Scalable Scheduling of Building Control Systems for Peak Demand Reduction

Truong X. Nghiem, Madhur Behl, Rahul Mangharam and George J. Pappas
Department of Electrical and Systems Engineering
University of Pennsylvania
{nghiem, mbehl, rahulm, pappasg}@seas.upenn.edu

***Abstract*— In large energy systems, peak demand might cause severe issues such as service disruption and high cost of energy production and distribution. Under the widely adopted peak-demand pricing policy, electricity customers are charged a very high price for their maximum demand to discourage their energy usage in peak load conditions. In buildings, peak demand is often the result of temporally correlated energy demand surges caused by uncoordinated operation of sub-systems such as heating, ventilating, air conditioning and refrigeration (HVAC&R) systems and lighting systems. We have previously presented *green scheduling* as an approach to schedule the building control systems within a constrained peak demand envelope while ensuring that custom climate conditions are facilitated. This paper provides a sufficient schedulability condition for the peak constraint to be realizable for a large and practical class of system dynamics that can capture certain nonlinear dynamics, inter-dependencies, and constrained disturbances. We also present a method for synthesizing periodic schedules for the system. The proposed method is demonstrated in a simulation example to be scalable and effective for a large-scale system.**

I. Introduction

Balancing the energy utilization of a large system is fundamental for their efficient behavior, especially in electrical systems and the electric grid [1]. *Peak demand* (or *peak load*), i.e., the largest simultaneous energy demand by all users in a system, might cause many severe issues such as low quality of service and service disruption, high cost of energy production and distribution, and so forth. For this reason, various technical and mostly economical methods have been used to control the peak demand, with the most widely adopted being *peak-demand pricing* [2]. In a peak-demand pricing policy, a large commercial electricity customer is charged not only for the amount of electricity it has consumed but also for its maximum demand over the billing cycle. The unit price of the peak demand charge is usually very high, up to 240 times in some cases [3] and even more, to discourage energy usage under peak load conditions. In other words, peaks in energy usage are inefficient and expensive for both suppliers and customers.

Building systems consist of many sub-systems such as heating, ventilating, air conditioning and refrigeration (HVAC&R) systems, boiler/chiller systems, and lighting systems. These components are often operated in an uncoordinated manner, i.e., independently of each other, which may result in temporally correlated power demand surges and in turn cause expensive electricity cost under the peak-demand pricing policy. As a result, demand control is important for the efficient operation of building control systems. Not only it helps reducing the electricity cost, it also has other benefits such as improving the quality of energy distribution and smoothing and flattening the curve of power usage.

While there exist several different approaches to load balance power consumption, e.g., by load shifting and load shedding [4], [5], they operate on coarse grained time scales and do not guarantee any thermal comfort. Another approach to energy efficient control for commercial buildings is model predictive control (MPC) ([6], [7]). In [6] the authors investigated MPC for thermal energy storage in building cooling systems. Peak electricity demand reduction by MPC with real-time pricing was considered in [7].

The problem of scheduling real-time computing tasks under resource constraints has been well studied over the past few decades in real-time systems [8]. In [9] and [10] the authors investigated the integration of control and real-time scheduling, in which control tasks were specified as periodic tasks with fixed execution time or CPU utilization and conventional real-time scheduling approaches were applied. From the resource allocation aspect, our control scheduling problem is similar to multiprocessor real-time scheduling with full migration [11], however the tasks' periods and execution times are highly dependent on the system's dynamics, the safety specifications, and the current state instead of being given a priori. Recently, a control scheduling problem for peak power reduction has been considered in [12], [13], in which independent control systems with affine dynamics and no disturbances are scheduled so that at most 1 actuator can be activated at any time and each state variable is bounded in a given range. However, these works are limited to simple dynamics and do not consider any disturbances to the system.

In our recent paper [14], we described an approach called *green scheduling* to reduce the peak demand of a large number of heating systems. We proposed a more general constraint that at most $k \geq 1$ actuators can be activated at any time. The system model considered in the paper was affine dynamics with no interaction between the sub-systems

This material is based upon work supported by the Greater Philadelphia Innovation Cluster (GPIC) for Energy Efficient Buildings an energy innovation HUB sponsored by the Department of Energy under Award Number DE-EE0004261.

and no disturbances. The main contribution of this paper is twofold.

1) The results in [14] are extended to system dynamics that are bounded between monotone affine dynamics. This class of dynamics is larger and more practical because it can capture certain nonlinear dynamics, inter-dependencies, and constrained disturbances.
2) We propose a scalable method for synthesizing periodic schedules for large-scale systems.

This paper is structured as follows. First, we formulate the system model and the problem in Section II. Section III proves a sufficient schedulability condition. Based on this proof, a method for synthesizing periodic schedules is described in Section IV. It is followed by a large-scale simulation in Section V which demonstrates the scalability and effectiveness of the proposed method. Finally, we conclude the paper with a road map of our future work in Section VI.

II. System Model

We present in this paper a scheduling approach to reduce the peak power demand of a heating system of multiple zones. Consider $n > 1$ zones. Each zone is heated by a heater that can be turned on, when it provides a constant heat input rate to the zone, and turned off, when it consumes no energy and provides no heat input. Let $x_i \in \mathbb{R}$ denote the air temperature (°C) of zone i and $q_i \in \mathbb{R}^+$ the heat input rate (kW) of the heater in that zone. Thermal comfort specifications require that x_i should be between a lower temperature threshold l_i and an upper temperature threshold $h_i > l_i$, i.e., x_i should be bounded in the range $[l_i, h_i]$. For zone i, let $T_{a,i}$ be its ambient air temperature (°C), which can be different for different zones, and d_i its internal heat gain (kW) from, e.g., its occupants. We consider $T_{a,i}$ and d_i as disturbances and define $w_i = [T_{a,i}, d_i]^T$ the disturbance vector of zone i. The law of conservation of energy gives us the following heat balance equation for zone i:

$$C_i \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = K_i \left(T_{a,i}(t) - x_i(t)\right) + \sum_{j \neq i} K_{ij} \left(x_j(t) - x_i(t)\right) + q_i(t) + d_i(t) \quad (1)$$

in which $C_i > 0$ is the thermal capacity of the zone (kJ/K), $K_i > 0$ the thermal conductance between the ambient air and the zone (kW/K), $K_{ij} \geq 0$ the thermal conductance between zone i and zone $j \neq i$ (kW/K).

The control input to heater i is its on/off state, denoted by $u_i \in \{0, 1\}$ where $u_i = 0$ corresponds to the off state and $u_i = 1$ the on state. Then its instant heat input rate is

$$q_i(t) = Q_i u_i(t) = \begin{cases} 0 & \text{if } u_i(t) = 0 \\ Q_i & \text{if } u_i(t) = 1 \end{cases} \quad (2)$$

for some constant $Q_i > 0$. Define $x = [x_1, \ldots, x_n]^T \in X$, where $X \subset \mathbb{R}^n$ is the value space of x. Since x_i is the temperature of a zone, it cannot receive any real value but only those in a valid range, for example between 15 °C and 30 °C. Therefore, X is a bounded subset of $\mathbb{R}^n$. From equations (1) and (2), the dynamics of x_i is governed by

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \begin{cases} f_{\mathrm{off},i}(x(t), w_i(t)) & \text{if } u_i(t) = 0 \\ f_{\mathrm{on},i}(x(t), w_i(t)) & \text{if } u_i(t) = 1 \end{cases} \quad (3)$$
$$x_i(0) = x_{0,i}$$

in which

$$f_{\mathrm{off},i}(x(t), w_i(t)) = \textstyle\sum_{j=1}^{n} A_{ij} x_j(t) + B_i w_i(t)$$
$$f_{\mathrm{on},i}(x(t), w_i(t)) = \textstyle\sum_{j=1}^{n} A_{ij} x_j(t) + Q_i / C_i + B_i w_i(t)$$
$$A_{ii} = -\left(K_i + \textstyle\sum_{j \neq i} K_{ij}\right) / C_i, \quad A_{ij} = K_{ij} / C_i \text{ for } j \neq i$$
$$B_i = [K_i / C_i, 1 / C_i], \quad x_{0,i} \in [l_i, h_i] \text{ is initial temperature.}$$

We assume that the dynamics of x_i is monotone: x_i always grows when $u_i = 1$ and always decays when $u_i = 0$. Hence, a zone's temperature always increases when its heater is on and always decreases when off.

A. Peak demand reduction problem

At any time t, the aggregate demand Q of the entire system is the sum of the power demands of individual heaters: $Q(t) = \sum_{i=1}^{n} Q_i u_i(t)$. As mentioned in Section I, under a demand-based tariff, the high charge for peak demand is an incentive for reducing the peak demand over a given time horizon $[0, t_f]$, to save energy and to reduce cost. However, we must also maintain the thermal comfort in each zone i, which requires that $x_i \in [l_i, h_i]$. Therefore, the peak demand reduction problem can be stated as follows.

Peak demand reduction problem: *Compute control inputs $u_i(t)$ for the heaters to minimize the peak demand $\max_{0 \leq t \leq t_f} Q(t)$ while maintaining thermal comfort in each zone.*

Conventional demand management strategies such as load shifting and load shedding ([4], [5]) can be used for this problem but they operate on coarse grained time scales and do not guarantee thermal comfort. Model predictive control ([6], [7]) is a powerful control framework for this problem, however its high computational requirement prevents it from being used for large-scale systems (with hundreds of zones and heaters).

B. Green scheduling for peak demand reduction

In [14], we proposed *green scheduling* as an approach to reduce the peak demand by coordinating the heaters so that at any time, at most k of them, for $1 \leq k \leq n$, can be on simultaneously while maintaining thermal comfort. The results were obtained for systems with no interaction between zones ($K_{ij} = 0$), no internal heat gain ($d_i = 0$), and a constant global ambient air temperature ($T_{a,i} = T_a = \text{const}$). In this paper, we remove those restrictions and derive a schedulability condition for the system as well as a *scalable* method for synthesizing periodic schedules.

Buildings in practice are typically well thermally insulated between zones and between the interior and the ambient air, i.e., K_i and K_{ij} are small. Furthermore, the disturbance w_i of a zone is usually constrained in some small bounded subset

$W_i \subset \mathbb{R}^2$. Therefore, it is reasonable to assume that the disturbances in each zone and the interactions between zones are small enough so that the temperature dynamics of each zone is bounded between two affine dynamics, independently of the other zones and the disturbances. This assumption is stated formally as follows.

Assumption 1 (Affinely bounded monotone dynamics): For each zone i, there exist $\underline{a}_{\mathrm{off},i} \geq 0$, $\overline{a}_{\mathrm{off},i} \geq 0$, $\underline{a}_{\mathrm{on},i} \geq 0$, $\overline{a}_{\mathrm{on},i} \geq 0$, $\underline{b}_{\mathrm{off},i}$, $\overline{b}_{\mathrm{off},i}$, $\underline{b}_{\mathrm{on},i}$, $\overline{b}_{\mathrm{on},i}$ such that for all $x \in X$ and all $w_i \in W_i$,

$$-\underline{a}_{\mathrm{off},i}x_i + \underline{b}_{\mathrm{off},i} \leq f_{\mathrm{off},i}(x, w_i) \leq -\overline{a}_{\mathrm{off},i}x_i + \overline{b}_{\mathrm{off},i} < 0$$

and $0 < -\underline{a}_{\mathrm{on},i}x_i + \underline{b}_{\mathrm{on},i} \leq f_{\mathrm{on},i}(x, w_i) \leq -\overline{a}_{\mathrm{on},i}x_i + \overline{b}_{\mathrm{on},i}$.

Given a schedule[1] $u(t) = [u_1(t), \ldots, u_n(t)]^T$ and the disturbance for each zone i, the trajectory of the system is the function $x : \mathbb{R}^+ \to X$ that satisfies the differential equation (3) for each i. A schedule $u(t)$ is *safe* if for each i, $x_i(t) \in [l_i, h_i]$ for all $t \geq 0$, i.e., thermal comfort is maintained in all zones. The system is said to be k-*schedulable*, for $1 \leq k \leq n$, if there exists a safe schedule $u(t)$ such that $\|u(t)\|_1 \leq k$ for all $t \geq 0$, i.e., at most k heaters can be on simultaneously.

We present a sufficient k-schedulability condition in the next section and a scalable method for synthesizing periodic schedules in Section IV.

Remark 1: Assumption 1 captures a larger class of systems than the heating dynamics in equation (3), including certain nonlinear dynamics. Because the results in the rest of this paper only use the parameters of the bound dynamics, they also hold for all systems belonging to this class.

III. Schedulability Analysis

The following theorem states a sufficient condition for the system to be k-schedulable.

Theorem 1: If for each i, $l_i < x_i(0) < h_i$ and $\underline{\eta}_i < \overline{\eta}_i$ where

$$\underline{\eta}_i = \frac{\underline{a}_{\mathrm{off},i}l_i - \underline{b}_{\mathrm{off},i}}{(\underline{a}_{\mathrm{off},i}l_i - \underline{b}_{\mathrm{off},i}) - (\underline{a}_{\mathrm{on},i}l_i - \underline{b}_{\mathrm{on},i})} \quad (4)$$

$$\text{and} \quad \overline{\eta}_i = \frac{\overline{a}_{\mathrm{off},i}h_i - \overline{b}_{\mathrm{off},i}}{(\overline{a}_{\mathrm{off},i}h_i - \overline{b}_{\mathrm{off},i}) - (\overline{a}_{\mathrm{on},i}h_i - \overline{b}_{\mathrm{on},i})} \quad (5)$$

then the system is k-schedulable for all $k > \underline{\eta} = \sum_{i=1}^n \underline{\eta}_i$, $k \in \mathbb{N}$.

Because $-\underline{a}_{\mathrm{off},i}l_i + \underline{b}_{\mathrm{off},i} < 0$ and $-\underline{a}_{\mathrm{on},i}l_i + \underline{b}_{\mathrm{on},i} > 0$ (Assumption 1), $\underline{\eta}_i$ is well-defined and satisfies $0 < \underline{\eta}_i < 1$. Similarly, $\overline{\eta}_i$ is well-defined and $0 < \overline{\eta}_i < 1$.

The rest of this section will present the proof of Theorem 1, which forms the basis of the scheduling synthesis in Section IV. We first define the following bound systems.

Definition 1 (Lower-bound system): For each i, define the lower-bound system $\underline{S}_i$ with state $\underline{x}_i \in \mathbb{R}$, control input $\underline{u}_i \in \{0, 1\}$, and dynamics given by

$$\frac{\mathrm{d}\underline{x}_i(t)}{\mathrm{d}t} = \begin{cases} -\underline{a}_{\mathrm{off},i}\underline{x}_i(t) + \underline{b}_{\mathrm{off},i} & \text{if } \underline{u}_i(t) = 0 \\ -\underline{a}_{\mathrm{on},i}\underline{x}_i(t) + \underline{b}_{\mathrm{on},i} & \text{if } \underline{u}_i(t) = 1 \end{cases} \quad (6)$$

[1]In this paper, we use the terms *control input* and *schedule* interchangeably for $u(t)$.

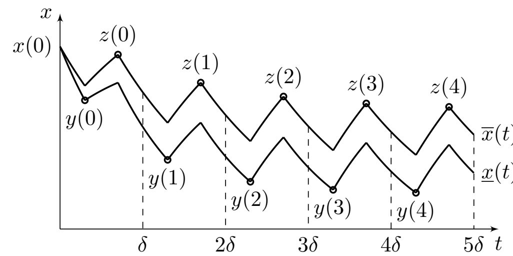


Fig. 1. Trajectories of the bound systems in Lemma 3.

Definition 2 (Upper-bound system): For each i, define the upper-bound system $\overline{S}_i$ with state $\overline{x}_i \in \mathbb{R}$, control input $\overline{u}_i \in \{0, 1\}$, and dynamics given by

$$\frac{\mathrm{d}\overline{x}_i(t)}{\mathrm{d}t} = \begin{cases} -\overline{a}_{\mathrm{off},i}\overline{x}_i(t) + \overline{b}_{\mathrm{off},i} & \text{if } \overline{u}_i(t) = 0 \\ -\overline{a}_{\mathrm{on},i}\overline{x}_i(t) + \overline{b}_{\mathrm{on},i} & \text{if } \overline{u}_i(t) = 1 \end{cases} \quad (7)$$

The following result is straightforward, which bounds the system's trajectory between those of the lower-bound and upper-bound systems.

Lemma 2: Given any schedule $u(t)$ and disturbances $\{w_i(t)\}_{i=1}^n$. For each i, the trajectory $x_i(t)$ is bounded by

$$\underline{x}_i(t) \leq x_i(t) \leq \overline{x}_i(t), \quad \forall t \geq 0$$

where $\underline{x}_i(\cdot)$ and $\overline{x}_i(\cdot)$ are respectively the trajectories of the lower-bound system $\underline{S}_i$ and the upper-bound system $\overline{S}_i$ with the same initial condition $\underline{x}_i(0) = \overline{x}_i(0) = x_i(0)$ and the same control $\underline{u}_i \equiv \overline{u}_i \equiv u_i$.

It follows that if the trajectories of $\underline{S}_i$ and $\overline{S}_i$ are bounded in $[l_i, h_i]$ then so is $x_i(t)$. The next lemma states the existence of safe periodic schedules for each heater i.

Lemma 3: Suppose $l_i < x_i(0) < h_i$. For any η_i such that $\underline{\eta}_i < \eta_i < \overline{\eta}_i$ and any $r_i \geq 0$, there exists $\delta_i^\star > 0$ such that if $u_i(t)$ is a periodic schedule

$$u_i(t) = \begin{cases} 1 & \text{if } (j + r_i)\,\delta \leq t < (j + r_i + \eta_i)\,\delta,\ j \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

for any δ satisfying $0 < \delta < \delta_i^\star$, then $x_i(t) \in [l_i, h_i], \forall t \geq 0$.

Proof: For the sake of clarity, we drop the subscript i in this proof. First, note that η always exists since $0 < \underline{\eta} < \overline{\eta} < 1$. Let $\underline{x}(t)$ and $\overline{x}(t)$ be the corresponding trajectories of the bound systems as defined in Lemma 2. Define $y(j) = \underline{x}((j + r)\delta)$ and $z(j) = \overline{x}((j + r + \eta)\delta)$ for $j \in \mathbb{N}$. Figure 1 illustrates these bound trajectories and the sequences $\{y(j)\}_{j\in\mathbb{N}}$ and $\{z(j)\}_{j\in\mathbb{N}}$. It follows from the monotonicity of the dynamics (Assumption 1) that

$$\inf_{t\geq 0} \underline{x}(t) = \inf_{j\in\mathbb{N}} y(j) \quad (9)$$

$$\sup_{t\geq 0} \overline{x}(t) = \max\left\{x(0), \sup_{j\in\mathbb{N}} z(j)\right\} \quad (10)$$

By Lemma 2, $\inf_{t\geq 0} \underline{x}(t) \leq x(t) \leq \sup_{t\geq 0} \overline{x}(t), \forall t \geq 0$.

Infimum of $\underline{x}(t)$: From (6), the sequence $\{y(j)\}_{j\in\mathbb{N}}$ is given by

$$y(0) = \mathrm{e}^{-\underline{a}_{\mathrm{off}} r\delta} x(0) + \frac{\underline{b}_{\mathrm{off}}}{\underline{a}_{\mathrm{off}}}\left(1 - \mathrm{e}^{-\underline{a}_{\mathrm{off}} r\delta}\right) \quad (11)$$

$$y(j + 1) = A_y(\delta)y(j) + B_y(\delta), \quad j \in \mathbb{N} \quad (12)$$

where

$$A_y(\delta) = \mathrm{e}^{-\delta\left(\underline{a}_{\mathrm{off}}(1-\eta)+\underline{a}_{\mathrm{on}}\eta\right)} \tag{13}$$

$$B_y(\delta)=\mathrm{e}^{-\underline{a}_{\mathrm{off}}(1-\eta)\delta}\left[\frac{\underline{b}_{\mathrm{on}}}{\underline{a}_{\mathrm{on}}}\left(1-\mathrm{e}^{-\underline{a}_{\mathrm{on}}\eta\delta}\right)-\frac{\underline{b}_{\mathrm{off}}}{\underline{a}_{\mathrm{off}}}\right]+\frac{\underline{b}_{\mathrm{off}}}{\underline{a}_{\mathrm{off}}} \tag{14}$$

Because $\delta > 0$, $\underline{a}_{\mathrm{off}} > 0$, $\underline{a}_{\mathrm{on}} > 0$ and $0 < \eta < 1$, $0 < A_y(\delta) < 1$. From linear system theory [15], the sequence $\{y(j)\}_{j\in\mathbb{N}}$ is monotonic and asymptotically converges to $y_\infty(\delta) = B_y(\delta)/(1 - A_y(\delta))$. Therefore

$$\inf_{t\geq 0} \underline{x}(t) = \inf_{j\in\mathbb{N}} y(j) = \min\{y(0), y_\infty(\delta)\} \tag{15}$$

Using straightforward calculus and algebra calculations, we can show that $y(0) \geq l$ for all $0 < \delta < -\frac{1}{\underline{a}_{\mathrm{off}} r}\ln\frac{l-\underline{b}_{\mathrm{off}}/\underline{a}_{\mathrm{off}}}{x(0)-\underline{b}_{\mathrm{off}}/\underline{a}_{\mathrm{off}}}$, and that $\lim_{\delta\to 0^+} y_\infty(\delta) > l$. Therefore, there exists $\delta_y > 0$ such that $0 < \delta < \delta_y$ implies $\inf_{t\geq 0} \underline{x}(t) \geq l$.

Supremum of $\overline{x}(t)$: Similarly, the sequence $\{z(j)\}_{j\in\mathbb{N}}$ is monotonic and asymptotically converges to $z_\infty(\delta) = C_z(\delta)/(1 - A_z(\delta))$ where

$$A_z(\delta) = \mathrm{e}^{-\delta(\overline{a}_{\mathrm{off}}(1-\eta)+\overline{a}_{\mathrm{on}}\eta)} \tag{16}$$

$$C_z(\delta)=\mathrm{e}^{-\overline{a}_{\mathrm{on}}\eta\delta}\left[\frac{\overline{b}_{\mathrm{off}}}{\overline{a}_{\mathrm{off}}}\left(1-\mathrm{e}^{-\overline{a}_{\mathrm{off}}(1-\eta)\delta}\right)-\frac{\overline{b}_{\mathrm{on}}}{\overline{a}_{\mathrm{on}}}\right]+\frac{\overline{b}_{\mathrm{on}}}{\overline{a}_{\mathrm{on}}} \tag{17}$$

Therefore

$$\sup_{j\in\mathbb{N}} z(j) = \max\{z(0), z_\infty(\delta)\} \tag{18}$$

Again, it can be shown that there exists $\delta_z > 0$ such that $0 < \delta < \delta_z$ implies $\sup_{t\geq 0} \overline{x}(t) \leq h$. Let $\delta^\star = \min\{\delta_y, \delta_z\} > 0$. Then $0 < \delta < \delta^\star$ implies $\inf_{t\geq 0} \underline{x}(t) \geq l$ and $\sup_{t\geq 0} \overline{x}(t) \leq h$, hence $l \leq x(t) \leq h$ for all $t \geq 0$. ∎

We now prove Theorem 1. To prove k-schedulability, we will construct a safe δ-periodic schedule $u(t)$ for the system so that at any time t, $\|u(t)\|_1 \leq k$. The time period $\delta > 0$ is chosen so that for every i, $x_i(t) \in [l_i, h_i]$ for all $t \geq 0$.

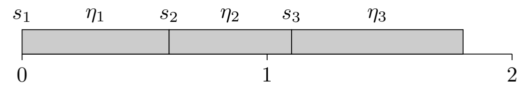
a) Constructing periodic schedules: We can always find $\eta_i > 0$ for each i such that $\underline{\eta}_i < \eta_i < \overline{\eta}_i$ and $\sum_{i=1}^n \eta_i \leq k$ because $0 < \underline{\eta}_i < \overline{\eta}_i < 1$ and $\sum_{i=1}^n \underline{\eta}_i < k$. We then distribute n non-overlapping right-open intervals, each of length η_i respectively, into the interval $[0, k]$ on the real line (Fig. 2a). Let interval i be $[s_i, s_i+\eta_i) \subseteq [0, k]$. Since $\sum_{i=1}^n \eta_i \leq k$, such a distribution is always possible. Given a time period δ, we construct the periodic schedule u_i as in (8) where $r_i = s_i - \lfloor s_i \rfloor \geq 0$ and $\lfloor s_i \rfloor$ denotes the largest integer that is no greater than s_i. Figure 2b illustrates this schedule construction for $n = 3$ and $k = 2$. It is straightforward to show that with these schedules, $\|u(t)\|_1 \leq k$ for all t.

b) Choosing δ: For each i, by Lemma 3, there exists $\delta_i^\star > 0$ such that schedule $u_i(t)$ is safe with any period δ satisfying $0 < \delta < \delta_i^\star$. Choose $0 < \delta < \min\{\delta_i^\star : i = 1, \ldots, n\}$. Then with this δ, the schedule $u(t)$ is safe, i.e., $x_i(t) \in [l_i, h_i]$ for all $t \geq 0$ and all i.

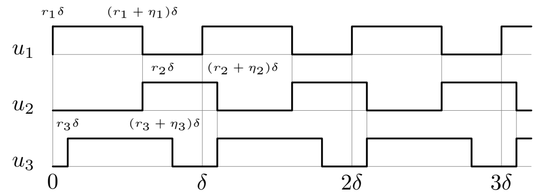
Therefore, the system is k-schedulable.

IV. Periodic Scheduling Synthesis

Suppose that the conditions in Theorem 1 are satisfied, thus the system is k-schedulable. The proof in Section III suggests that periodic schedules of the form (8) can be used for the system. However, to make these schedules safe, their time periods might need to be chosen very small (Lemma 3). In practice, this is usually undesirable, even impossible, because of the physical constraints of the actuators or the performance degradation of the actuators caused by high-frequency switching. Therefore, to consider practical periodic scheduling, we need to relax the safety requirements:



(a) Distribution of n non-overlapping intervals into $[0, k]$.



(b) Timing diagram of the constructed schedules u_i from interval distribution (a): $r_1 = s_1$, $r_2 = s_2$, $r_3 = s_3 - 1$.

Fig. 2. Construction of safe periodic schedules for $n = 3$ and $k = 2$. At any time, at most $k = 2$ control inputs are ON simultaneously.

$$\text{for each } i,\ x_i(t) \in [l_i, h_i] \quad \forall t \geq \tau_i$$

where $\tau_i \geq 0$ is finite. In other words, x_i is allowed to be out of the comfort range for a finite time horizon $[0, \tau_i)$, and after that it must be bounded in the range.

Each periodic schedule $u_i(t)$ in (8) has three parameters: time period δ_i, utilization η_i, and offset r_i. Since η_i is the fraction of the time period that $u_i = 1$, we borrow the term *utilization* from real-time scheduling [8]. We denote the set of parameters of schedule u_i by (δ_i, η_i, r_i). For each i, $x_i(t)$ is bounded between $\underline{x}_i(t)$ and $\overline{x}_i(t)$ (Lemma 2), whose bounds are given by (9), (10), (15), and (18). Since the safety requirements have been relaxed, we only need to consider the limits $y_{\infty,i}(\delta_i, \eta_i)$ and $z_{\infty,i}(\delta_i, \eta_i)$ calculated as

$$y_{\infty,i}(\delta_i, \eta_i) = B_{y,i}(\delta_i, \eta_i)/(1 - A_{y,i}(\delta_i, \eta_i))$$
$$z_{\infty,i}(\delta_i, \eta_i) = C_{z,i}(\delta_i, \eta_i)/(1 - A_{z,i}(\delta_i, \eta_i))$$

where $A_{y,i}$, $B_{y,i}$, $A_{z,i}$ and $C_{z,i}$ are defined in (13), (14), (16), and (17). Observe that these limits depend on the values of δ_i and η_i, but not r_i. Therefore, we can construct periodic schedules u_i in two steps: (1) for each i, compute δ_i and η_i to make the limits bounded in $[l_i, h_i]$; (2) given parameters (δ_i, η_i) for all i, find r_i so that at any time t, $\|u(t)\|_1 \leq k$.

A. Step 1: Compute (δ_i, η_i) for each i

In this step we compute δ_i and η_i for each i so that $y_{\infty,i}(\delta_i, \eta_i) \geq l_i$ and $z_{\infty,i}(\delta_i, \eta_i) \leq h_i$. In practice, the time period δ_i is determined by the characteristics of the physical equipments and the hardware platform. Thus, we assume that $\delta_i > 0$ is provided and we need to compute η_i.

By taking the derivative of $y_{\infty,i}$ with respect to η_i, it is straightforward to verify that $\frac{\mathrm{d}y_{\infty,i}}{\mathrm{d}\eta_i} > 0$ for all $0 < \eta_i < 1$ and $\delta_i > 0$, i.e., the function $y_{\infty,i}$ is strictly increasing with

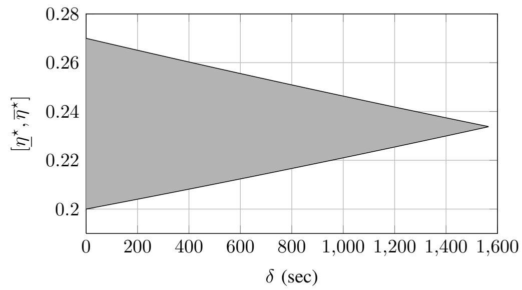


Fig. 3. Feasible region $(\delta_i, [\underline{\eta}_i^\star, \overline{\eta}_i^\star])$.

respect to η_i. It follows that $y_{\infty,i}(\eta_i) \geq l_i$ is equivalent to $\eta_i \geq \underline{\eta}_i^\star$ where $\underline{\eta}_i^\star$ is the root of the equation $y_{\infty,i}(\underline{\eta}_i) = l_i$. Though we do not have a closed-form expression for $\underline{\eta}_i^\star$, the equation can be numerically solved efficiently using Newton's method since $y_{\infty,i}(\eta_i)$ is strictly monotonic. Similarly, the constraint $z_{\infty,i}(\eta_i) \leq h_i$ is equivalent to $\eta_i \leq \overline{\eta}_i^\star$ where $z_{\infty,i}(\overline{\eta}_i^\star) = h_i$, which can also be solved numerically.

If $\underline{\eta}_i^\star \leq \overline{\eta}_i^\star$ then we can choose any value η_i in the range $[\underline{\eta}_i^\star, \overline{\eta}_i^\star]$. Otherwise, if $\underline{\eta}_i^\star > \overline{\eta}_i^\star$, the given time period is infeasible and δ_i needs to be reduced. Indeed, there exists a maximal feasible time period $\delta_i^\star$, for which $\underline{\eta}_i^\star = \overline{\eta}_i^\star$, that can be computed numerically. Figure 3 illustrates the feasible region $(\delta_i, [\underline{\eta}_i^\star, \overline{\eta}_i^\star])$ (gray-filled) for system parameters $\underline{a}_{\text{on},i} = \underline{a}_{\text{off},i} = \overline{a}_{\text{on},i} = \overline{a}_{\text{off},i} = 0.00025$, $\underline{b}_{\text{on},i} = 0.013$, $\underline{b}_{\text{off},i} = 0.003$, $\overline{b}_{\text{on},i} = 0.0133$, $\overline{b}_{\text{off},i} = 0.0033$, $l_i = 20$, and $h_i = 24$. In this case, $\delta_i^\star$ is about $1565\,\text{s}$.

B. Step 2: Compute r_i

In this step, given (δ_i, η_i) for all i and $k \geq \sum_{i=1}^n \eta_i$, $k \in \mathbb{N}$, we compute $\{r_i\}_{i=1}^n$ so that the periodic schedule defined in (8) satisfies $\|u(t)\|_1 \leq k$, $\forall t \geq 0$. In general, this problem is similar to multiprocessor real-time scheduling of periodic tasks with full migration [11], in which δ_i is the task's period, η_i is the task's utilization, and k is the number of identical processors. Conventional multiprocessor scheduling algorithms can be used to derive a schedule for the system. However, if the time periods δ_i are uniform (i.e., $\delta_i = \delta$ for all i), there exists a simple algorithm for obtaining the values r_i as shown in Section III. This algorithm is simple and scalable for a large value of n, however it requires that all schedules u_i have the same time period.

C. Safety guarantee

Let $X_{0,i}$ be the set of initial states of x_i. Using the periodic schedule (8) with parameters $\{(\delta_i, \eta_i, r_i)\}_{i=1}^n$, we can find a finite time horizon τ_i for each i such that $x_i(t)$ is guaranteed to be in the range $[l_i, h_i]$ for all $t \geq \tau_i$. Indeed, since the sequences $\{y_i(j)\}_{j\in\mathbb{N}}$ and $\{z_i(j)\}_{j\in\mathbb{N}}$ (Section III) that bound the trajectory $x_i(t)$ are monotonic and converge to $y_{\infty,i}$ and $z_{\infty,i}$ respectively, once they are in $[l_i, h_i]$ they will stay in that range indefinitely. Therefore, τ_i corresponds to the smallest time step j such that $y_i(j) \geq l_i$ and $z_i(j) \leq h_i$ for all initial state $x_i(0) \in X_{0,i}$. It can be easily calculated from the expressions of $y_i(j)$ and $z_i(j)$.

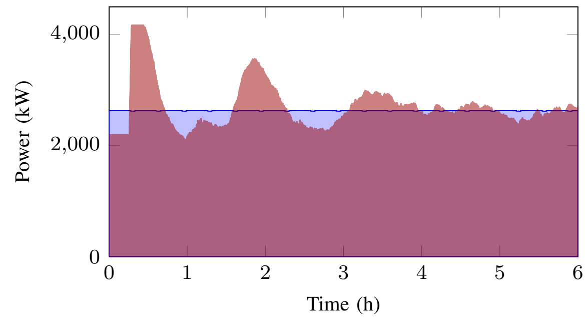


Fig. 4. Power demand of the periodic schedule with $\delta = 20\,\text{min}$ (blue) and the uncoordinated On-Off control (red).

TABLE I

PEAK DEMANDS P (MW) AND ENERGY CONSUMPTIONS E (MW h).

	Time Period $\delta = 20\,\text{min}$		Time Period $\delta = 10\,\text{min}$	
	On-Off	Periodic	On-Off	Periodic
P	4.173	2.629	4.173	2.563
E	16.167	15.766	16.167	15.364

V. SIMULATION

In this section we present simulation results and compare the proposed approach to uncoordinated On-Off control for large scale systems. The *periodic schedule* described in Section IV was implemented in MATLAB.

We considered 500 zones whose parameters were randomly generated with zone's thermal capacity $C_i \in [2000, 3000]$ (kJ/K) and thermal conductance K_i chosen proportionately from $[0.4, 0.6]$(kW/K). The thermal capacity of a zone is an indicator of the size of the zone, so a greater value of C_i corresponds to a larger zone. The zones are heated by heaters with different heat input rates q_i also chosen based on the size of the zone $q_i \in \{7, 10\}$(kW). Since the number of zones was large, we randomly assigned zones which can thermally interact with each other and the value of their inter-zonal thermal conductance $K_{i,j}$ was chosen from $[0, 0.06]$ (kW/K), with the value 0 implying that the zones do not interact.

Zone temperatures were required to be kept between $l = 20\,°\text{C}$ and $h = 24\,°\text{C}$. The ambient air temperature profile was different for every zone and varying over time with the value being bounded in $[10, 12]$ (°C). The disturbances d_i due to internal heat gain from occupants, appliances, etc. were also different for every zone and time-varying.

The simulation time was 6 hours. We ran two simulations with different values of the time period δ of the schedule and compared the peak demand and total energy consumption with the uncoordinated On-Off control. MPC was not implemented for comparison since it could not be scaled to a large (500) number of zones. The peak demands and energy consumptions are reported in Table I.

A. Performance

Compared to the uncoordinated On-Off controller, the periodic scheduler significantly reduced the peak demand by about 38%. When the time period δ was $20\,\text{min}$, the value of k (*k-schedulable*) was 316 for 500 zones which resulted in

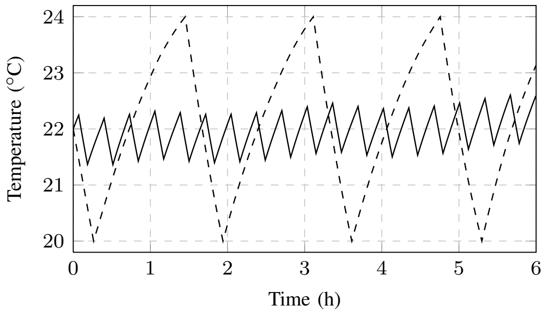


Fig. 5. Temperature profile for a single zone for periodic schedule (solid) and uncoordinated On-Off control (dashed) for $\delta = 20\,\text{min}$.

a 37.0% reduction in the peak power demand and a 2.49% decrease in the total energy consumption as compared to the uncoordinated case. The energy consumption for this case is shown in Figure 4. Obviously, the curve of energy usage of the periodic schedule was much more smooth and flat compared to that of the uncoordinated On-Off control. The periodic schedule computation ran very fast and took about 3.3 seconds to complete. When the time period δ was decreased by half from $20\,\text{min}$ to $10\,\text{min}$ we observed a greater reduction in the peak demand (38.58%). The value of k decreased from 316 (for $\delta = 20\,\text{min}$) to 308 for this case. The savings in the total energy consumption also increased to 4.97%. Since the time period was smaller, tasks switched more frequently but resulted in better savings. This can be seen in Figure 5 which shows the temperature profile for a single zone for the On-Off (shown in dashed) and the periodic case (shown in solid). Although the periodic scheduler switches more frequently than On-Off control, it maintains the temperature of the zone within a smaller range (around the mean temperature of $22\,^{\circ}\text{C}$), which is better with respect to both thermal comfort and the power consumption of the zone. The peak demand and energy consumption for the uncoordinated On-Off case remained the same in both cases as it did not depend on the value of the time period.

VI. Conclusion

We presented an approach to energy efficient control of building systems by scheduling them within a constrained peak demand envelope while maintaining the required environmental specifications. The class of system dynamics considered in this paper can include inter-dependencies between sub-systems, constrained internal and external disturbances, and certain nonlinear dynamics. A sufficient schedulability condition was derived and a method for synthesizing periodic schedules for the system was proposed. Through a large-scale simulation, the method was shown to be scalable as well as effective in reducing peak demand.

In this paper, we assumed the existence of independent bound dynamics that capture the inter-dependencies between sub-systems. Although this assumption is reasonable in practical applications, it might not hold when the inter-dependencies are large. In the future, we aim to remove this drawback by directly working with the inter-dependent dynamics of the system. We are also investigating state-feedback dynamic scheduling algorithms, dynamic pricing models, operational efficiency and task-specific cost functions for system-wide optimization.

Disclaimer

References

[1] H. Hermanns and H. Wiechmann, "Future design challenges for electric energy supply," in *Proc. IEEE Conf. Emerging Technologies & Factory Automation ETFA 2009*, 2009, pp. 1–8.

[2] M. H. Albadi and E. F. El-Saadany, "Demand response in electricity markets: An overview," in *Proc. IEEE Power Engineering Society General Meeting*, 2007, pp. 1–5.

[3] TRFund Study, "Understanding PECO's general service tariff," 2007.

[4] S. Kiliccote, M. Piette, and D. Hansen, "Advanced controls and communications for demand response and energy efficiency in commercial buildings," in *Carnegie Mellon Conf. Elec. Power Sys.*, 2006.

[5] K. ho Lee and J. E. Braun, "Development of methods for determining demand-limiting setpoint trajectories in buildings using short-term measurements," *Building and Environment*, vol. 43, no. 10, pp. 1755 – 1768, 2008.

[6] Y. Ma, F. Borrelli, B. Hencey, B. Coffey, S. Bengea, and P. Haves, "Model predictive control for the operation of building cooling systems," in *Proc. ACC'10*, 2010, pp. 5106–5111.

[7] F. Oldewurtel, A. Ulbig, A. Parisio, G. Andersson, and M. Morari, "Reducing peak electricity demand in building climate control using real-time pricing and model predictive control," in *Proc. IEEE CDC'10*, 2010, pp. 1927–1932.

[8] J. W. S. Liu, *Real-time Systems*. Prentice Hall, 2000.

[9] A. Cervin and J. Eker, "The control server: A computational model for real-time control tasks," in *Proc. ECRTS '03*, July 2003, pp. 113–120.

[10] R. Chandra, X. Liu, and L. Sha, "On the scheduling of flexible and reliable real-time control systems," *Real-Time Systems*, vol. 24, pp. 153–169, March 2003.

[11] R. I. Davis and A. Burn, "A survey of hard real-time scheduling algorithms and schedulability analysis techniques for multiprocessor systems," University of York, Tech. Rep., 2009.

[12] M. D. Vedova, M. Ruggeri, and T. Facchinetti, "On real-time physical systems," in *Proc. RTNS'10*, 2010, pp. 41–49.

[13] T. Facchinetti, E. Bibi, and M. Bertogna, "Reducing the peak power through real-time scheduling techniques in cyber-physical energy systems," in *First International Workshop on Energy Aware Design and Analysis of Cyber Physical Systems*, 2010.

[14] T. X. Nghiem, M. Behl, R. Mangharam, and G. J. Pappas, "Green scheduling of control systems for peak demand reduction," in *IEEE Conference on Decision and Control (CDC 2011)*, December 2011.

[15] W. J. Rugh, *Linear System Theory*, 2nd ed. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1996.